A STATISTICAL METHOD of IDENTIFYING
GENERAL BUCKLING MODES on the CHINOOK
HELI.COPTER FUSELAGE

by

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Abstract:
A method for determining general buckling modes is presented. Test cases are used to demonstrate the process. The method has been successfully used on the CH-47 Chinook helicopter program at Boeing Philadelphia to identify global fuselage buckling modes.

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Introduction:
The design engineer must ensure his design meets the performance goals established. To achieve this, the engineer must consider the structural behavior during all conditions. In addition to performance, the structure must be able to maintain margin against all modes of failure. These service and safety concerns include, but are not limited to: excessive deflection or vibration, stress failures, corrosive and fatigue failures, and buckling failures.

This paper will concentrate on buckling failures. For purposes of this paper, we identify two types of buckling: local (panel) buckling and global (general) buckling.

Local buckling is characterized by a small portion of the structure buckling. Examples are skin wrinkling or tertiary struts buckling. While there are documented cases of local buckling leading to catastrophic failures, many structures behave with no service degradation if some portions of the structure are buckled. In the case of a helicopter fuselage, it is common to design a lightweight skin that is allowed to buckle while taking the vibration and inertial loads through a dedicated structural space frame. There is no degradation in flight performance, and the local skin buckling rarely leads to a catastrophic failure.

Global buckling is characterized by the entire structure (or a large portion of the structure) undergoing buckling. Often this buckling is catastrophic.

To predict the stability behavior of structures, the Finite Element Method provides a computerized method for calculating theoretical buckling for math model representations. Buckling is solved using an Eigensolution method using the differential stiffness based on the loading and boundary conditions. For complex assemblies, the Eigensolution results cannot necessarily be interpreted from the numeric Eigenvalue. To interpret the results, the engineer must have visualization of the Eigenvectors from the buckling solution. While most engineers can agree to which modes are “global” and which are “local,” the process is subjective, time consuming, and cannot be used to automate a design optimization process. This is especially true in aerospace structures when trying to separate “global” buckling modes from “local” panel modes.

Hence, the question, “How can I identify which buckling mode is a global mode without having to manually look at the deflected shape of every mode?” The purpose of this paper is to provide a statistical approach to identify global modes.

1. Theory
Determining “global” modes is different in dynamics than buckling. Although both employ an Eigensolution to determine the modes and mode shapes, they differ in the properties of the modes. For instance, in dynamics, the normal modes exhibit special properties if mass normalized. With a normal mode, one can calculate “participation factors” and “modal effective mass.” Unfortunately, no such properties have been identified for buckling Eigensolutions.
The method for quantifying global modes in this paper is based on statistics. MSC.Nastran calculates Eigenvectors for each buckling mode. One property of the Eigenvectors is that the modes are normalized so that the maximum analysis set displacement of the shape is set to 1.0. The statistical approach will take advantage of this property.

The Statistical Mean of a Buckling Eigenvector:

\[ \mu_x = \frac{1}{n} \sum_{i=1}^{n} x_i \]

For a FEM model with a reasonable mesh distribution, a “local” mode will have a relatively few dof with “high” Eigenvector displacement values. For example, with the maximum displacement set to 1.0, a local mode will only have a very small percent of displacements within 2 orders of magnitude of 1.0. So a low mean defines a local mode. Conversely, a “higher” mean (say 0.1 or higher) indicates a significant portion of the structure is involved in the Eigenvector.

The Statistical Standard Deviation of a Buckling Eigenvector:

\[ \sigma_x = \sqrt{\frac{1}{n-1} \sum_{i=1}^{n} (x_i - \mu_x)^2} \]

A similar argument for the mean can be made for the standard deviation. A low standard deviation indicates a local mode, and a higher standard deviation indicates a global mode.

The Weighted Standard Deviation of a Buckling Eigenvector:

\[ \sigma = \sigma_x \cdot \mu_x \]

While the mean and standard deviation are indicators of global modes, they cannot be looked at individually. Numeric test cases have shown that some modes with “high” means are, in fact, local modes. But when the mean and standard deviation are considered together, a definite trend emerges. The “weighted standard deviation,” is defined in this paper as the mean*std_dev. Because the numbers are between 0.0 and 1.0, the modes that cannot be deemed “local” or “global” by examining the mean or standard deviation alone can be classified using the weighted standard deviation.

Method: The statistical method for identifying global buckling modes developed in this paper uses the following algorithm

1. Convert Eigenvectors into the BASIC coordinate system to allow averaging in the same direction.
2. Separate the Eigenvectors into TRANSLATIONAL COMPONENTS (because a high rotation would indicate a local mode for real world structures)

3. Manipulate Eigenvectors such that each term is positive. The user has the option of using the absolute value of the Eigenvector, or squaring the Eigenvector. (For example, if this is not done then the second Euler buckling mode of a simply supported column will be identified as a “local” mode because the mean will be 0.0)

4. Optionally reduce the Eigenvectors to a subset of “hard-points” prior to statistical computations.

5. Perform statistics in each manipulated component direction (X, Y, and Z) and, optionally, perform statistics on the magnitude of the components.

6. Print results.

**2. Test Case**

Consider the stiffened panel shown in Figure 1.

![Image of stiffened panel](image_url)

Figure 1. Stiffened panel test case

The MSC.Nastran model was used to extract the first 100 buckling modes, and the statistical approach above was applied. Representative buckling modes are shown in Figure 2. Four different statistical options are documented in Figures 3 thru 6.
Figure 2. Buckling Modes 1, 15, 21, 42, and 57 (in ascending order left to right)

Figure 3. Statistics Results using Absolute Value of Eigenvector

Figure 4. Statistical Results using Square of Eigenvector
Of the 100 modes calculated, only 3 were deemed “global” based on Eigenvector plots. Mode 15 is the first Euler buckling global mode, Mode 42 is the first torsion mode, and Mode 57 is the second order Euler mode. The other modes computed were similar to local Mode 1, where only a few panels were active in the Eigenvector. The statistical approach had trouble eliminating Mode 21 when using the entire model, but it drops out when using a reduced set of GRIDs. It is very similar to Mode 15, but with more local panels moving.

Of the four statistical approaches presented, it is clear that limiting the statistical calculations to a subset of the entire model consistent with primary load carrying members helped to identify the global modes more clearly. Figure 3, statistics on the absolute value of entire Eigenvector, indicates difficulty in clearly distinguishing between global and local modes. Figure 4, statistics on the square of the entire Eigenvector, provides an order of magnitude separation between global and local modes. But the data used to generate Figures 5 and 6 (statistics on the square of a subset of the Eigenvectors) provides clear separation of at least 4 to 6 orders of magnitude between the local modes and global modes except for Modes 21 and 71. In the case of Mode 21, there are 2 orders of magnitude between it and the other 3 global modes.

Based on the statistical analysis of the Eigenvectors for the test case, the following generalizations can be made about the method:
• Squaring the Eigenvector prior to statistical computations is advisable. The rationale for doing this is that the squared terms are similar to an energy calculation.

• Limiting the GRIDs for statistical computation is clearly advantageous when identifying global buckling modes (i.e. using GRIDs associated with hard points or primary structural members).

• When applying the above two options, the “global” Eigenvectors can be identified easily with approximately 2 orders of magnitude between the global and local modes when comparing the weighted standard deviation.

3. Application to Chinook Fuselage

Background:
As part of a trade study, it is desired to study the general stability of the Chinook center fuselage using a MSC.Nastran model. This study will focus on different material systems and substructure layouts, so the ability to rapidly evaluate the general stability eigenvalue of the entire fuselage is imperative to effectively traversing the design space. The area of interest is shown in Figure 7:

The Chinook is a tandem-rotor craft- there is a forward and aft rotor. This arrangement subjects the fuselage to a large vertical bending moment that puts the top of the vehicle in compression, which is where the buckling failure should occur. The fuselage structure is comprised of thin skins that are allowed to fully buckle at limit load, a sequence of full and half frames, and several longerons. The basic structural arrangement is shown in figure 8:
Figure 8: Structural Arrangement of the Chinook fuselage

Approach:
In order to determine how well the statistical method identifies general buckling modes, the design load condition with the maximum fuselage vertical bending moment is used. Since the design criteria does not rely on the skin, there are several “local” modes that must be calculated along with the primary member modes. In general, we do not know what eigenvalue range to use prior to an analysis, or during subsequent optimization cycles, so an extremely large range of eigenvalues is used. Any modes with an eigenvalue above the cutoff are considered non-critical for the loading. The statistical method will then identify which buckling modes correspond to general buckling modes. These are then inspected by hand to determine the first general mode of the fuselage. The statistical method results are then compared to the manual result for validation.

It is not desirable to manually sort through all of the mode shapes in order to find the first stability eigenvalue. The only robust manner of doing this is to look at the mode shape of each result, which could easily mean thousands of mode shapes. However, it is necessary for this study because it will validate or disprove the statistical method.

Definition of General Buckling:
The criteria for determining the first critical mode of the fuselage is based on Ref. 2:

In general instability, failure is not confined to the region between two adjacent frames or rings but may extend over a distance of several frame spacings… In panel instability, the transverse stiffeners provided by the frames on rings is sufficient to enforce nodes in the stringers at the frame support points…

The concept of general buckling is further illustrated in figure 9:
For this study, general instability of the Chinook fuselage will be defined as a mode shape in which the buckled shape includes two full frames deflecting in the same direction. In addition, the magnitude of these eigenvectors should be sufficient enough to indicate a realistic stability mode.

**Critical Load Condition:**

The design condition that applies the maximum vertical bending moment to the fuselage is used in this study. This load is applied at the forward and aft rotor hubs and reacted by inertial relief on the fuselage. A running load plot of the maximum vertical bending moment condition is shown in figure 10:
The loading shown is a symmetric load condition which should produce a failure on the roof of the fuselage.

Manual Iteration on Eigenvalue:

The best model to find the critical stability mode is shown in figure 10. This model is the best finite element representation of the Chinook, and has approximately 120,000 degrees of freedom.

However, since the results of this model will be manually sorted to validate the statistical method, a simplified model with approximately 18,000 degrees of freedom was used for this study, as shown in figure 12. This model has much larger skin elements and the frames are represented solely as bar elements.
This model also greatly reduced the CPU time needed to run the model. Ideally, the coarse grid model will be used to rapidly identify global buckling modes, while the fine grid model will be used to validate the results.

The critical load case was run using MSC.Nastran solution 105. For this model, it is known that the failure will occur on the top of the fuselage, and since the load condition is symmetric, it is possible to interrogate only the centerline frame nodes of the model; greatly reducing the domain of the manual sorting problem.

To manually find a stability mode that matches the criteria for a general mode, results for the major frames along the roof centerline were dumped to an Excel spreadsheet. Vertical displacements of the nodes along the fuselage centerline were plotted to allow visual scanning for a general mode.

Using this technique, the first critical mode number is 701. Due to the sensitive nature of the Chinook’s design information, the eigenvalue for this mode is not included in this paper. The mode shape, and a plot of the centerline deflection along the major frames is shown in figure 13:
Figure 13: Mode Shape 701 – First general mode of fuselage
Due to the monotony of inspecting 700+ modes by hand, this approach required over three days worth of work. In addition, it is possible that a critical mode was missed.

Statistical Method results:

The same simplified model was also run using the statistical method for determining global modes. All of the frame nodes on the upper part of the fuselage were used to compute the statistical measure. Another benefit of this method is a much shorter run time of the model, since no results need to be requested. The first 1000 modes of the model yielded the statistical measures shown in figure 14:

![Figure 14: Statistical Measures of mode 0-1000](image)

The first mode number that is visible on this graph is mode 701, which matches the hand sorting method. The amount of time to arrive at this mode using the statistical approach was much less than an hour.
Conclusions

1. A statistical method for identifying global buckling modes was presented. This approach was developed to enable an analyst to quickly determine the global buckling modes of a structure.

2. The method was validated on a simple test case. By using a model that was fully understood, the method was able to correctly identify the global buckling mode. In addition, this test model was used to demonstrate the effect of using a reduced set of nodes to calculate the statistical measures.

3. The method was validated on a real world model of the Chinook helicopter. The first global mode of the Chinook helicopter was determined by manual sorting of the MSC.Nastran results, then used to verify the statistical method. The two techniques yielded the same result.

4. The method showed time savings of three days to one hour. The comparison of the methods done on the Chinook model showed how much time the statistical method can save. This savings occurs because the analyst does not have to produce or manipulate large amounts of data.

5. The ability to specify the area of interest yields more accurate results. Both the test case and the Chinook model demonstrated that the global modes become more evident when a set of nodes is used to compute the statistical measures.

Limitations and Caveats. In order to truly leverage the functionality of this method, it is important that the end-user have a general understanding of the structure’s response to loading. This understanding also allows the analyst to use a reduced set of nodes in the area of interest. Finally, the results of this method should always be scrutinized, to exclude any significant modes that do not represent the true structures response to loaded (for example, modes due to modeling practices, etc…).

4. Further Work

1. Integrate method into MSC.Nastran solution 200. The ability to quantitatively identity significant buckling modes of a structure allows a numerical optimizer to use these modes a design response. Ideally, this functionality would be incorporated into MSC.Nastran Solution 200(5), to allow for a comprehensive optimization constrained by global stability.
2. Allow measures to be computed for multiple sets. This functionality will allow the analyst to identify the global mode for several individual parts of a model. This would be a quick way to determine the stability of local regions and the overall model at the same time.

3. Patran integration. To further accelerate the process, Boeing is developing a MSC.Patran interface to this method. This will allow the user to identify the area of interest, run the model, and evaluate results, all within the same comprehensive framework.

4. Expand validation case using Chinook fine-grid model. The coarse-grid model of the Chinook was used to improve runtime and allow the results to manually sorted. Because the statistical method has been validated on this coarse grid model, and due to the time savings that it gives us, it needs to be evaluated on the fine grid model. Success here indicates that this method is valid.

5. References


Appendix A- DMAP listings for MSC.Nastran V2001

$ PURPOSE: Determine "global" buckling modes
$ METHOD: statistics on eigenvectors
$ PARAMETERS:
$  STATTYPE: [Default=2]
  Manipulation of eigenenvectors prior to statistics
  1 = Absolute Value of Eigenvectors
  2 = Square Eigenvectors (Recommended)
$  SMTERMS: [Default=.001]
  Filter for small terms prior to statistics
$  STATCOMP: [Default='YES']
  Perform statistics in component directions
$  STATRES: [Default='YES']
  Perform statistics on resultant (Tx Ty Tz)
$ for SOL 105
malter 'call.*super3.*bug'
type parm,,rs,y,stattype=2. $ 1 for absolute value, 2 for square
type parm,,rs,y,smterms=.001$
type parm,,char3,y,statcomp='YES'
type parm,,char3,y,statres='YES'

$ convert bug (buckling displacement G-size) to basic
VECPLT BUG,BGPDT,SCSTM,CSTMS,CASEcc,,/BUGBtmp/
/0/1/ PARAM,'/OUICORD,'/BASIC '/ALTSHAPE//SEID $
$ partition down to "statistically retained set"
$ setid must be 105105
matmod eqexins,uset,sils,casecc,,/vecset/17/7341583/-105105/
s,n,setok////////'BE+BF+C+''R+Q+O+SB''+SG+M' $
IF (setok=-1) THEN
  EQUIVX BUGBtmp/BUGB/ALWAYS $
ELSE
  PARTN BUGBtmp,,vecset,,BUGB,,/1 $
ENDIF $

PARAML BUGB//'TRAILER'/1/S,N,ugcols$
PARAML BUGB//'TRAILER'/2/S,N,ugrows$

$ strip out translational dof only one at a time
matgen ,/trvecx/4/1/ugrows/0/1/6/1/1/1 $
matgen ,/trvecy/4/1/ugrows/0/1/6/2/1/1 $
matgen ,/trvecz/4/1/ugrows/0/1/6/3/1/1 $
partn bugb,,trvecx,,bugbx/1 $ bugbx = basic X displ's for each mode
partn bugb,,trvecy,,bugby/1 $ numgrid rows X numeig cols
partn bugb,,trvecz,,bugbz/1 $
IF (statcomp='YES') THEN
$ stats on BASIC components of GSET displacements
    call statsx bugbx/meanx,varx,stddevx,wtstdevx/
        stattype/smterms/'X'/TRUE $
    call statsx bugby/meany,vary,stddevy,wtstdevy/
        stattype/smterms/'Y'/TRUE $
    call statsx bugbz/meanz,varz,stddevz,wtstdevz/
        stattype/smterms/'Z'/TRUE $
ENDIF

IF (statres='YES') THEN
$ calculate magnitude of displacements (must use GSET vectors)
    diagonal bugbx/bugbx2/'WHOLE'/2.
    diagonal bugby/bugby2/'WHOLE'/2.
    diagonal bugbz/bugbz2/'WHOLE'/2.
    add5 bugbx2,bugby2,bugbz2,,/sumsqrs $
    diagonal sumsqrs/resltnt/'WHOLE'/.5 $
$ stats on magnitude of displacements -- Grid size
    call statsx resltnt/meanR,varR,stddevR,wstdevR/
        stattype/smterms/'RSLT'/TRUE $
ENDIF $

include 'statistics.dmap'
endalter

DMAP LISTING 1, SIGMODES_PARTN.DMAP
(concluded)
$ purpose: compute statistical mean, variance and standard deviation $ on column vector(s) $ $ INPUT: $ INVEC column vector(s) $ OUTPUT: $ MEAN -- mean column vector(s) $ VARIANCE -- variance of column vector(s) $ STDEV -- standard deviation column vectors(s) $ PARAMETERS: $ VMANIP (REAL) $ =1. take absolute value of column vector(s) prior to stats $ =2. square column vectors(s) prior to stats $ any other value then do not modify input vectors $ SMTERMS1 (REAL) $ >0. filter small terms from manipulated column vector(s) prior to stats $ <=0. do not filter small terms $ STATTYPE (CHARACTER*4) $ used for printing headers $ SQFLAG (LOGICAL) $ TRUE -- multiply weighted stdev by square of max term $ FALSE - unaltered weighted stddev $ $ message //'stats on vector(s) tagged as '/stattype/’ direction' $ IF (vmanip=1.) THEN $ message//'using absolute value of vector(s) prior to stats' ELSE IF (vmanip=2.) THEN $ message//'squaring vector(s) prior to stats' ELSE $ message//'using raw vector(s) for statistics' ENDIF $ IF (smterms1>0.) THEN $ message //'/filtering terms smaller than '/smterms1 $ ELSE $ message //'/no filtering prior to statistics' ENDIF $ DMAP LISTING 2, STATX.DMAP (continued below) IF (vmanip=1. or vmanip=2.) THEN
diagonal invvec/invecm/'WHOLE'/vmanip $ 1=abs value, 2=square
ELSE
   equivx invvec/invecm/always$
ENDIF

$ filter absolute value vectors
IF (smterms1>0.) THEN
   matmod invvecm,,,,,,/invecmf,/2///smterms1
ELSE
   equivx invvecm/invecmf $
ENDIF $

$ get unit matrix for summation
PARAML invecmf//'TRAILER'/2/S,N,numrowsi$
IF (numrowsi<2) THEN
   message//'dmap info message' $
   message//'after filtering matrix for direction '/stattype/',' $
   message//'it is too small to provide' $ 
   paraml invvec//'TRAILER'/1/s,n,numcols
   matgen ,/meanmat/7/numcols/1 $
   matgen ,/variance/7/numcols/1 $
   matgen ,/stddev/7/numcols/1 $
   RETURN $ if null vector after filter, then return
ENDIF $
matgen ,/umat1/6/numrowsi/0/numrowsi $

$ summation of all terms in each column vector
mpyad invvecmf,umat1,/summat/1 $

$ calculate mean = 1/n * (summat)
numrowsr=real(numrowsi)
numrowsc=CMPLX(1./numrowsr)
add summat,/mean/numrowsc $

$ calculate variance
$ create xbar vector (fill rows with mean value)
trnsp mean/meant
mpyad umat1,meant,xbar 
add invvecmf,xbar/xdiff/(1.,0.)/(-1.,0.)/0 $ (xi - mean)
diagonal xdiff/xdiffsg/'WHOLE'/.2. $ (xi - mean)^2
mpyad xdiffsg,umat1,/xdifs/1 $ sum (xi - mean)^2
nminus1=CMPLX(1./(numrowsr-1.)) $ (1/(n-1))
add xdifs,/variance/nminus1 $ 1/(n-1) * sum (xi-mean)^2

$ calculate stddev (=variance^2)
diagonal variance/stddev/'WHOLE'/.5 $

$ calculate weighted standard deviation = mean * stddev
add mean,stddev/wtstdev///1

DMAP LISTING 2, STATX.DMAP (continued below)

IF (sqflag) THEN
$ get maximum values of each column
matmod invecmf,./maxvals,/7 $
  diagonal maxvals/maxvals2/'WHOLE'/2. $ square the terms
$    message //'modifying weighted standard deviation '/
$    'by the following values:'
$    matprn maxvals2//$
    add maxvals2,wtstddev/wtdstddev///1 $
ELSE
  equivx wtstddev/wtdstddev/always
ENDIF
message //'mean values for each vector in direction '/
  stattype/'follow:' $
matprn mean //$
message //'standard deviation for each vector in direction '/
  stattype/'follow:' $
matprn stddev //$
message //'weighted standard deviation for each vector in direction '/
  stattype/'follow:' $
matprn wtdstddev //$
return $
end $