The Equivalent Analysis of Honeycomb Sandwich Plates for Satellite Structure

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Abstract: The honeycomb sandwich plate is applied widely in the modern satellite design because of its excellent capability. When a satellite structure analyzed by MSC.Nastran, the equivalent parameters of the honeycomb sandwich plates had to be identified. In this paper three equivalent methods had been studied, that is called the sandwich theory, the honeycomb-plate theory and the equivalent-plate theory. Through the three methods the natural frequencies of a honeycomb sandwich plate including two load cases had been calculated. The computational results show that the three equivalent methods discussed in this paper are reliable and practical in the finite element analysis of satellite structures.

1. Introduction
The honeycomb sandwich plate is popular in the aviation, spaceflight region, especially in the satellite design. A honeycomb sandwich plate consists of the top surface layer, the honeycomb core and the bottom surface layer, shown in figure 1. According to its projective shape, the honeycomb cell can be sort into hexagon, diamond, rectangle and so on, in which the hexagon honeycomb is the most popular because of its simpler production and higher efficiency. In this paper only the hexagon honeycomb is investigated, shown in figure 2.

Fig.1 Honeycomb sandwich plate     Fig.2 Hexagon honeycomb cell

It’s well known that the equivalent parameters of the honeycomb sandwich plate had to be achieved before analyzed by MSC.Nastran. In this paper three equivalent methods, that is the sandwich theory, the honeycomb-plate theory and the equivalent-plate theory, were studied. For the first one only the honeycomb core is equalized, while for the last two the whole honeycomb sandwich plate is equalized.

2. The equivalent parameters’ identification of the honeycomb sandwich plate
2.1 The sandwich theory
It is assumed that the core can resist of the transverse shearing deformation and has some in-plane stiffness, while the top and bottom surface layer cannot resist of the shearing deformation but satisfy the Kirchhoff hypothesis. Under the above assumption the honeycomb core can be regarded as an orthotropic layer. For the hexagon honeycomb core, the equivalent elastic parameters are as follows:
\[ E_x = E_y = \frac{4}{\sqrt{3}} \left( \frac{t}{l} \right)^3 E \quad G_{xy} = \frac{\sqrt{3} \gamma}{2} \left( \frac{t}{l} \right)^3 E \]

\[ G_{xx} = \frac{\gamma}{\sqrt{3} l} G \quad G_{yy} = \frac{\sqrt{3} \gamma}{2} \frac{t}{l} G \quad \nu_{xy} = 1/3 \]

Where \( E, G \) are the engineering constants of the material of the core; \( l, t \) are the length and thickness of the honeycomb cell; \( \gamma \) is the technology corrected coefficient whose value is about between 0.4 and 0.6.

In MSC.Patran the material property of the whole honeycomb sandwich can be established, shown in figure 3, in which mat1 indicates the material of top and bottom surface layer and core’s material parameters are the equivalent values obtained from the above formulae.

![Fig. 3 The sandwich theory used in MSC.Patran](image)

### 2.2 The honeycomb-plate theory

In this section both the honeycomb sandwich plate and the equivalent plate were analyzed with the REDDY low-order shear deformation theory. The honeycomb sandwich plate and the equivalent plate with the same size are shown in figure 4.

![Fig. 4 Honeycomb sandwich plate (left) and its equivalent plate (right)](image)
The displacement on the cross section is continuous, so according to the low-order shear deformation theory, the displacement of the cross section has to satisfy the following equations:

\[
\begin{align*}
\mu(x, y, z, t) &= z \Phi_x(x, y, t) \\
\nu(x, y, z, t) &= z \Phi_y(x, y, t) \\
\omega(x, y, t) &= \omega(x, y, t)
\end{align*}
\]

According to the Hamilton theory, it can be known that the dynamic equations of the honeycomb sandwich plate and the equivalent plate own the same form. Through the stiffness equalization the elastic parameters of the equivalent plate can be obtained, and the mass density achieved from the inertia equalization. The equivalent parameters are as follows:

\[
\begin{align*}
\bar{E}_x &= \left( e_{11}e_{22} - e_{12}^2 \right)/e_{22} \\
\bar{E}_y &= \left( e_{11}e_{22} - e_{12}^2 \right)/e_{11} \\
\bar{G}_{xz} &= e_{44} \\
\bar{G}_{yz} &= e_{55} \\
\bar{G}_{xy} &= e_{66} \\
\bar{v}_{xy} &= e_{12}/e_{22}
\end{align*}
\]

Where

\[
\begin{align*}
e_{11} &= \frac{\left( (h+d)^3 - h^3 \right) e_{f11} + h^3 e_{111}}{(h+d)^3} \\
e_{12} &= \frac{\left( (h+d)^3 - h^3 \right) e_{f12} + h^3 e_{122}}{(h+d)^3} \\
e_{22} &= \frac{\left( (h+d)^3 - h^3 \right) e_{f22} + h^3 e_{222}}{(h+d)^3} \\
e_{44} &= \frac{d}{h+d} e_{f44} + \frac{h}{h+d} e_{e44} \\
e_{55} &= \frac{d}{h+d} e_{f55} + \frac{h}{h+d} e_{e55} \\
e_{66} &= \frac{\left( (h+d)^3 - h^3 \right) e_{f66} + h^3 e_{666}}{(h+d)^3}
\end{align*}
\]

\[
\rho = \rho_f + \frac{h}{h+d} \rho_e
\]

and

\[
\begin{align*}
e_{c11} &= e_{c22} = \frac{1}{1 - \nu^2} \bar{E}_x \\
e_{c12} &= \frac{\nu}{1 - \nu^2} \bar{E}_x \\
e_{c44} &= e_{c55} = G_{xz} = G_{yz} \\
e_{c66} &= G_{xy}
\end{align*}
\]

\[
\begin{align*}
e_{f11} &= e_{f22} = \frac{1}{1 - \nu^2} E \\
e_{f12} &= \frac{\nu}{1 - \nu^2} E \\
e_{f44} &= e_{f55} = KG \\
e_{f66} &= G
\end{align*}
\]

Where \( \nu \) is the poisson ratio of the material of the top and bottom surface layer, \( d \) is the thickness of the surface layer; \( h \) is one half of the thickness of the core. \( e_{fi}, e_{eij} \) are respectively the stiffness parameters of the material of the surface layer and the core under the above coordinate frame. \( \rho_f, \rho_e \) are the mass density of the surface layer and the core respectively. \( k \) is the effect
2.3 The equivalent-plate theory

The whole thickness of the honeycomb sandwich plate is $2H$, and for the top and bottom surface layer, the thickness is $t$, the elastic modulus is $E$ and the poisson ratio is $\mu$. Assumed that the Kirchhoff hypothesis is met, the bending stiffness of the equivalent plate is $\frac{Et^3}{12(1-\mu^2)}$.

According to the bending stiffness equivalent between the honeycomb sandwich plate and the equivalent plate, the following equation can be achieved:

$$\frac{E_{eq}t_{eq}^3}{12(1-\mu^2)} = \frac{2E}{1-\mu^2} \left( \frac{t^3}{12} + \frac{(H-t)^2}{2} \right) \quad (1)$$

Where $t_{eq}$, $E_{eq}$, $\mu$ are the thickness, the elastic modulus, and the poisson ratio of the equivalent plate, respectively.

In the same way it can be deduced from the equivalence of the axial stiffness between the honeycomb sandwich plate and the equivalent plate:

$$E_{eq}t_{eq} = 2Et \quad (2)$$

From the equation (1) and (2),

$$t_{eq} = \sqrt{t^2 + 12h^2} \quad E_{eq} = \frac{2Et}{t_{eq}}$$

Where $h = H - t/2$.

According to the mass equivalence, the equivalent mass density $\rho_{eq}$ can be achieved.

$$\rho_{eq}t_{eq} = 2\rho_1t + 2\rho_2(H - t)$$

Where $\rho_1$ is the mass density of the surface layer and $\rho_2$ is the mass density of the core. Therefore

$$\rho_{eq} = \frac{2\rho_1t + 2\rho_2(H - t)}{t_{eq}}$$

3. The example

The finite element model of a honeycomb sandwich plate which is from a satellite had been established by using MSC.Patran, shown in figure 5. The thickness of the top, bottom aluminum surface layer is 0.3mm; the core whose thickness is 24.4mm is made up of the hexagon cell whose length is 4mm, thickness is 0.04mm. For the aluminum material, the elastic modulus is 70GPa and the poisson ratio is 0.3.

In this paper the honeycomb sandwich plate is subject to two load cases which have different boundary conditions. In the load case one the four corner points can rotate freely but their transverse
displacements are zero; while in the load case two all the transverse and the rotational displacements of the four corner points are zero. For each load case the natural frequencies are calculated under four different finite element mesh, that is $4 \times 8$, $10 \times 20$, $50 \times 100$ and $100 \times 200$. The computation results are shown in table 1, in which method 1 indicates the sandwich theory, method 2 is the honeycomb-plate theory and method 3 is the equivalent-plate theory.

It can be found that the results of method 2 are uppermost and those of method 1 are lowermost. All the three equivalent methods can meet the demands of the finite element analysis. Moreover, it is worth noting that the computation results were affected obviously by the boundary conditions and even with the same boundary conditions in the load case two, the finite element mesh is a sensitive factor in the vibration analysis.

![Fig. 5 The finite element model of the honeycomb sandwich plate](image)

<table>
<thead>
<tr>
<th>Equivalent method</th>
<th>Mode number</th>
<th>Load case one</th>
<th>Load case two</th>
</tr>
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</table>
|                   | $4 \times 8$| $10 \times 20$| $50 \times 100$| $100 \times 200$| $4 \times 8$| $10 \times 20$| $50 \times 100$| $100 \times 200$
| Method 1 | 1 | 41.96 | 41.30 | 40.91 | 40.79 | 77.58 | 66.94 | 56.78 | 54.27 |
|              | 2 | 155.3 | 147.3 | 142.5 | 141.0 | 198.2 | 176.7 | 160.5 | 155.9 |
| Method 2 | 1 | 42.34 | 41.84 | 41.71 | 41.69 | 83.03 | 72.73 | 60.89 | 57.54 |
|              | 2 | 164.5 | 157.7 | 156.1 | 155.9 | 222.6 | 196.9 | 180.6 | 176.5 |
| Method 3 | 1 | 42.04 | 41.51 | 41.34 | 41.32 | 81.40 | 70.75 | 59.11 | 56.46 |
|              | 2 | 164.5 | 155.0 | 152.8 | 152.5 | 216.3 | 192.0 | 176.0 | 172.1 |

**Reference**
