Abstract

In MSC.Nastran the aerodynamic matrix is calculated and manipulated to form generalized aerodynamic influence coefficients (AIC). The AIC matrix can be perceived as a complex stiffness matrix in the general equations of motion. The numerical condition of the aerodynamic matrix is not as fully evaluated by MSC.Nastran in the aeroelastic analysis solution sequences as it could be. A study is made to evaluate the numerical behavior of the aerodynamic matrix for both subsonic and supersonic conditions using matrix tools from Version 2001 of MSC.Nastran. The aerodynamic matrix determinant value and the singular value decomposition terms are calculated and summarized for a few sample wing planform configurations.
A Study of Aerodynamic Matrix Numerical Condition
By
Dean Bellinger and Tony Pototzky

Introduction

The aeroelastic capability of MSC.Nastran generates aerodynamic influence coefficient (AIC) matrices that can be thought of as complex aerodynamic stiffness matrices. The user’s guide by Rodden and Johnson\(^1\) describes the aeroelastic capability of MSC.Nastran. The AIC matrices are generated from aerodynamic matrices calculated by the subsonic Doublet-Lattice or the supersonic ZONA51 methods. The complex/unsymmetric AIC matrices do not allow the symmetric structural matrix-to-factor diagonal ratio to be used to measure the numerical condition. This study is performed to determine if the matrix determinant or the singular value decomposition (SVD) provides a means of measuring the numerical characteristics and condition of the aerodynamic matrices and determine the simulation suitability of the aerodynamic matrices. This latter item leads to the real purpose of this study: “Can the determinant or singular value decomposition identify when the AIC matrix becomes unusable for aeroelastic calculation?”

A series of typical planform configurations are investigated in an attempt to answer the above question and to determine the numerical condition/characteristic of the aerodynamic matrices. Both subsonic and supersonic conditions are investigated and summarized in this study. The planform parameters, e.g., sweep and taper, are also part of this study. A delta wing is investigated as the final planform configuration. Data for the determinant and SVD value variation with reduced frequency are presented to determine if they can be used to answer the above question.

Model Description

Aerodynamic surface planforms from Touvila and McCarty (NACA RM L55E11) were used in the study to investigate the determinant (Det) and singular value decomposition ratio (SVDR). The planforms consist of three basic configurations. The first planform configuration uses four models having constant chord and four sweep angles of 15, 30, 45 and 60 degrees. The second planform configuration uses two delta wings of 45 and 60 degrees of sweep. The third planform configuration combines sweep and taper, only two of the five configurations described by Touvila and McCarty were used. Of the tapered models chosen for this study, one has a taper ratio, \(\lambda\), of 0.2 and no sweep at the quarter-chord and the second has a taper ratio of 0.4 and 45 degrees of sweep. All models were examined at one subsonic and one supersonic Mach number. The models have a plane of symmetry as shown by the wing planforms in Figures 1 and 2.

The two planform figures list the Mach number, sweep and taper variations made to conduct this study. The 15° sweep model of Figure 1 is a well-publicized model used by Rodden and Johnson\(^1\) to perform static, flutter and response aeroelastic analyses. The aerodynamic model representation used in Rodden and Johnson\(^1\) has four chordwise (streamwise) and six spanwise elements. The aerodynamic element mesh for the constant chord aerodynamic model is one of six aerodynamic models used in this study. The other five aerodynamic models increment the
number of chordwise elements by four elements while maintaining the same element aspect ratio. The highest value of reduced frequency, \( k \), of Rodden and Johnson\(^1\) for the flutter and

![Figure 1](image1.png)

Figure 1. – Constant Chord Example Planforms with Four Sweep Angles and Two Mach Numbers.

![Figure 2](image2.png)

Figure 2. – Taper and Delta Wing Example Planforms at Two Sweep Angles, Two Mach Numbers and Two Taper Ratos.
response analyses are 0.20. This value of reduced frequency is well within the acceptable range according to Rodden and Johnson’s\textsuperscript{1} aerodynamic modeling guidelines in Section 3.1. This modeling rule has since been revised and reported by Rodden at the June 1999 Aerospace Flutter and Dynamics Council Meeting. The guideline of Rodden an Johnson\textsuperscript{1} yields the following equation:

\[ N_{cbox} = \left(\frac{12}{\pi}\right)k \]  \hspace{1cm} (1)

The revised guideline yields the following relationship of number of chordwise boxes to the reduced frequency.

\[ N_{cbox} = \left(\frac{48}{\pi}\right)k \]  \hspace{1cm} (2)

**Aerodynamic Matrix Processing**

Two approaches for evaluating the numerical behavior of the aerodynamic matrix are employed in this study. The determinant of the matrix is one way of learning about the aerodynamic matrix. Numerical difficulties are likely to be encountered during the decomposition of a matrix when the determinant tends to zero. However, this presents a problem because the more detail of the aerodynamic mesh, the determinant may grow very large or very small. In the cases investigated in this study, the determinant grew smaller with increasing mesh density. So a better method of evaluating the numerical condition of the aerodynamic matrix was sought for this study. One approach is available from the CEAD module of MSC.Nastran for Version 70.6 and later version. The approach is the Singular Value Decomposition of a matrix. Numerous references describe this approach and Golub and Van Loan\textsuperscript{3} provide valuable insight with regard to the determination of the numerical condition of a matrix. The determinant is not a viable measure of numerical condition; however, its behavior seems to indicate some puzzling evidence as shown later in this report. The SVD shows some valuable information about the ZONA51 aerodynamic matrix. Figures 3 through 6 display the actual maximum and minimum value.

![Figure 3. – Maximum Singular Value Decomposition Values for the Quartic-DLM Aerodynamic Matrix.](image-url)
output by the SVD value method for the aerodynamic matrices of the quartic-DLM and the ZONA51 methods. The maximum and minimum values were used to create a ratio of these two values for more convenient evaluation.

![Graph](image1.png)

**Figure 4.** – Minimum Singular Value Decomposition Values for the Quartic-DLM Aerodynamic Matrix.

![Graph](image2.png)

**Figure 5.** – Maximum Singular Value Decomposition Values for the ZONA51 Aerodynamic Matrix.
A method to process and present the Det and SVD data is required to fulfill this study. A DMAP alter is created for SOL 145, aeroelastic flutter analysis, to compute the Det and SVD values for the configurations shown in Figures 1 and 2. The Det and SVD values are normalized with respect to the Det and SVD values at a reduced frequency, k, of 0.001 as given in Equations 3 and 4.

\[
\text{Nrm(Det)} = \frac{\text{Det}(m,k)}{\text{Det}(m,0.001)} \tag{3}
\]

\[
\text{Nrm(SVDR)} = \frac{\text{SVD}(m,k)}{\text{SVD}(m,0.001)} \tag{4}
\]

The DMAP alter and typical input data file are presented in Listings 1 and 2 at the end of this report. The DMAP alter calculates and formats the Det and SVD values to facilitate importing into Microsoft Excel. Excel provides a general method of xy-plotting of results presentation and comprehension. The models were setup to output the Det and SVD values for all of the model configurations shown in Figures 1 and 2 above. The MKAERO1 entry used with each planform...
contained 84 \( k \) values and one Mach number to describe the \( \text{Det} \) and \( \text{SVD} \) value behavior versus \( k \). Normally, this quantity of reduced frequencies is not required to perform an aeroelastic analysis. A large number of \( k \) values is used to study the \( \text{Nrm}(\text{Det}) \) and \( \text{Nrm}(\text{SVDR}) \) variation with reduced frequency and ensure that some numerical anomaly is not overlooked as will be seen in some of the \( \text{SVDR} \) data. Results for each planform are determined for one subsonic and one supersonic Mach number.

**Results**

Table 1 summarizes the runs made to obtain the \( \text{Det} \) and \( \text{SVD} \) values. The 15 degree swept wing model has the most in-depth analysis of the suite of models. Note that this model is run with quadratic and quartic DLM for the six chordwise aerodynamic element variation. This model shows some interesting behavior of the \( \text{Det} \) values in the subsonic case. The supersonic case is run with the coarser mesh sets of aerodynamic elements. The tapered and delta wing configurations are run with the least number of chordwise aerodynamic elements except for Quartic DLM aerodynamics.

For discussion purposes, only the results of the 15-degree swept wing model will be presented. The \( \text{Det} \) and \( \text{SVDR} \) values are given in Figures 7 – 10 for subsonic case using quadratic and quartic DLM aerodynamics. The \( \text{Det} \) values in Figure 7 for the quadratic-DLM generated AIC’s tend to zero indicating a singular matrix at higher reduced frequencies. However, the \( \text{SVDR} \) values in Figure 8 for the same matrices indicate well-conditioned matrices making them inconsistent with the \( \text{Det} \). Nevertheless, for the same aero model, the \( \text{Det} \) values in Figure 9 from the quartic-DLM generated matrices indicate an opposite behavior of increasing with increasing frequency instead of going to zero. Again, the \( \text{SVDR} \) values in Figure 10 show a similar trend as Figure 8, especially for the greater aero mesh density. Figures 11 and 12 show the model using supersonic aerodynamics from the ZONA51 method. Interestingly, both \( \text{Det} \)

<table>
<thead>
<tr>
<th>Model Description</th>
<th>Sweep</th>
<th>Number of Chordwise Boxes</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>4</td>
</tr>
<tr>
<td>Constant Chord, i.e., no taper</td>
<td>15</td>
<td>1 x 2</td>
</tr>
<tr>
<td></td>
<td>30</td>
<td>1 x 2</td>
</tr>
<tr>
<td></td>
<td>45</td>
<td>1 x 2</td>
</tr>
<tr>
<td></td>
<td>60</td>
<td>1 x 2</td>
</tr>
<tr>
<td>Tapered</td>
<td>0</td>
<td>1 2</td>
</tr>
<tr>
<td></td>
<td>45</td>
<td>1 2</td>
</tr>
<tr>
<td>Delta</td>
<td>45</td>
<td>1 x 2</td>
</tr>
<tr>
<td></td>
<td>60</td>
<td>1 x 2</td>
</tr>
</tbody>
</table>

1- Quadratic DLM, x – Quartic DLM, 2 – ZONA51
and SVDR values of Figures 11 and 12 are consistent in showing singular matrices at approximately reduced frequencies of 7.3 and 14 for the 4 and 8 box cases, respectively. For comparison with the Det and SVDR value variation over reduced frequency, a generalized aerodynamic influence coefficient is given in Appendix A. The values shown in Appendix A are from the constant chord-15° swept wing model at subsonic and supersonic.

![Figure 7](image.png)

Figure 7. – Normalized Determinant of Quadratic DLM Aerodynamic matrix as a Function of Reduced Frequency.

![Figure 8](image.png)

Figure 8. – Normalized Singular Value Decomposition Ratio Variation with Reduced Frequency for Quadratic DLM Aerodynamics.
speeds for a rigid body pitch mode. The pitch mode generalized aerodynamic influence
coefficients for the 4 and 8 box supersonic cases in Figures A3 and A4 show the erratic
tendencies above the reduced frequency of 7 and near the reduced frequency of 14. These
tendencies are consistent with the results shown in Figures 11 and 12.

Figure 9. – Normalized Determinant Variation with Reduced Frequency for Quartic DLM Aerodynamics.

Figure 10. – Normalized Singular Value Decomposition Ratio Variation with Reduced Frequency with Quartic DLM Aerodynamics.
Admittedly, the range of reduced frequencies is purposely specified over an extreme range as seen in the above figures. While it is not necessary or prudent to perform a flutter or aeroelastic response analysis. The internal interpolation capabilities over the MKAERO1 reduced frequency range by the FA1 module means that a more frugal number of reduced frequencies can be use to determine the effect of any intermediate reduced frequency with any of the available flutter

Figure 11. – Normalized Determinant Variation with Reduced Frequency for Supersonic ZONA51 Aerodynamics.

Figure 12. – Normalized Singular Value Decomposition Ratio Variation for Reduced Frequency with ZONA51 Aerodynamics.
methods of the program. A large number of reduced frequencies are used in this study to ensure continuity of the Det and SVDR values. For practical reasons the reduced frequencies above 6.0 are more widely spread than below the 6.0 value.

Conclusions

More varied aerodynamic configurations need further study with additional mesh density to better understand the numerical characteristics.

SVD demonstrates the numerical condition of the aerodynamic matrices. DLM behavior appears to be without numerical difficulty. ZONA51 shows poor numerical condition for coarsest meshes above reduced frequencies of 6.0.

When using the Quartic-DLM method, the determinant value is relatively consistent in its behavior. Quadratic-DLM and ZONA51 methods are more erratic at the higher reduced frequencies and are planform dependent.

The SVDR values do not demonstrate any excessive values and the DLM matrices appear to be numerically well conditioned. The onset of the large amplitude variation of the determinant magnitude or the SVDR value only occurs after the guidelines established in Equation 2 are exceeded. If one applies the guideline then any numerical problems with the aerodynamic matrix is avoided. In answer to the question raised at the beginning of the report, in general, the determinant develops high values for the quartic-DLM at the higher reduced frequencies, but this is exhibited after violation of the guideline of Equation 2.

Acknowledgement

With the assistance of Tom Kowalski of MSC.Software Corporation, the singular value decomposition matrix was implemented as part of this study.

References

Listing 1. - DMAP Alter to Output Det and SVD Ratios

$ A DMAP alter to output Singular Value Decomposition
$ maximum/minimum matrix ratio, SVDR, values and the
$ Determinant of the AJJT matrix calculated from DLM
$ or ZONA51 aerodynamic methods.
$ by Dean Bellinger, October, 1999.
$ If CHKAJJT is set to YES with PARAM,CHKAJJT=YES in the
$ bulddata section of input then the DECOMP module is called
$ so that it deselects the SPARSE decomposition method. This
$ write the DET and POWER output parameters from the DECOMP
$ module to a USERFILE on Fortran unit 25. Additionally, a
$ parameter CEIGAJJT can be set to YES to calculate the
$ Singular Value Decomposition matrix from the CEAD module
$ so that the SVDR value can be calculated and output. The
$ SVDR values is also written to the userfile on Fortran unit 25.
$ However, the SVDR is a good measure of the numerical condition
$ of the AJJT matrix.

User Input:
File Management Section -
assign userfile='15d-045-24b.f25' unit=25 form=formatted delete new

Bulk Data -
PARAM,CHKAJJT,YES (default is NO)
PARAM,CEIGAJJT,YES (default is NO)

COMPILE PFAERO SOUIN=MSCSOU NOLIST NOREF
ALTER 'DECOMP.*AJJT','DECOMP.*AJJT'
TYPE PARM,,CHAR8,Y,CHKAJJT='NO      ' $ IF (CHKAJJT='YES') THEN $ CALL CHKAJJT AJJT, CASEAA / LAJJT, UAJJT /
ELSE $ DECOMP AJJT/LA JJT,UAJJT, $ ENDIF $ CHKAJJT

SUBDMAP CHKAJJT A,CASEAA/L,U/KBAR/MACHNO/KCNT $ $ TYPE DB,DYNAMICS $ TYPE PARM,,I,N,NOGOOD,BAD,KCNT $ TYPE PARM,,,RS,N,KBAR,MACHNO $ $ $ FIND COMPLEX EIGENVALUES OF AJJT $ IF (CEIGAJJT='YES     ') THEN $ compute the aero matrix Singular Value Decomposition values
CEAD A,,,, ,,/,CLAMA,,,svals/ S,N,WCEIGV//-1/'svd'//0 $ MESSAGE //SINGULAR VALUE DECOMP. FOR AJJT MATRIX AT K OF:'/KBAR $ OFP CLAMA // $ diagonal svals/svdiag/'column'/1.0 $ extract diagonal of svals
matmod svdiag/,svdiagm1/'whole'//-1.0 $ reciprocal of each svdiag terms
matmod sdiag1,,,,/svdmax,/,7 $ find maximum of svals diagonal terms
add sdiagmax,svdim1/svdrat///1 $ SVDR (should be 1x1 matrix)
matprn svdrat // $ paraml svdrat//'dmi'/1/1/s,n,svdrat $
$\text{MATPRN CPHDX,LPHDX // $ Uncomment for Eigenvector output}$

$\text{MATMOD A,,,,,/ACCT,,10 $}$

$\text{TRNSP ACC/ACCT}$

$\text{MPLYAD A,ACCT,,/AACCT///6 $}$

$\text{MATMOD AACCT,,,//RAACCT,IAACCT/34 $}$

$\text{PARAML RAACCT//TRAILER'1/S,N,NCRRAACCT}$

$\text{READ RAACCT,RIDENT,,,DYNAMICS,,CASEAA,,,,,/LAMRAACC}$

$\text{VRAACCT,NRAACCT,OEIGS,,/'MODES'/S,N,NERAACCT/1 $}$

$\text{OFFP LAMRAACC,OEIGS // $}$

$\text{paraml svdmax//'dmi'/1/1/s,n,svdmax}$

$\text{paraml svdminm1//'dmi'/1/1/s,n,svdmin}$

$\text{svdmin = 1.0/svdmin}$

$\text{ENDIF $ CEIGAJJT}$

$\text{DECOMPOSITION COMPUTE THE DETERMINANT}$

$\text{$ Setting of SYS209 to deactivate sparse method is required}$

$\text{$ to obtain the determinant of the Ajj matrix.}$

$\text{$ PUTSYS(0,209) $ DEACTIVATE SPARSE UNSYMMETRIC DECOMPOSITION}$

$\text{DECOMP A/L,U,,/-1//S,N,MINDIAG/S,N,DET/S,N,POWER/S,N,SING/}$

$\text{S,N,NBRCHG/S,N,MAXRAT}$

$\text{NOGOOD = 0-NBRCNG $}$

$\text{IF ( NOGOOD<0 OR SING<0 ) BAD=-1 $}$

$\text{IF (BAD = -1) THEN $}$

$\text{MESSAGE //THE AJJT MATRIX IS PROBABLY SINGULAR.'/}$

$\text{' THIS MAY BE CAUSED BY PLACE A PANEL ON'/}$

$\text{' A PLANE OF SYMMETRY.' $}$

$\text{$ PUTPRAMP // $ For debug output}$

$\text{EXIT $}$

$\text{ELSE $}$

$\text{$ Set up output so it is easy to import into Microsoft Excel}$

$\text{$ putsys(25,2) $ put following message in unit 25 file}$

$\text{$ MESSAGE //RED FREQ:'/KBAR/$}$

$\text{$ ' DET MAN:'/DET/' POW 10:'/POWER/$}$

$\text{$ ' SVD RAT:'/svdmax/svdmax/svdmin}$

$\text{$ 2 3 4 5 6 7 8 9}$

$\text{IF ( KCNT <= 2 ) MESSAGE // MACH NO. '/'$}$

$\text{' RED. FREQ.'/'}$

$\text{' RL(DET.MAN.)'/'}$

$\text{' IM(DET.MAN.)'/'}$

$\text{' POWER/' SVD Ratio'/'}$

$\text{' MAX SVD'/'}$

$\text{' MIN SVD '$}$

$\text{MESSAGE //MACHNO/KBAR/DET/POWER/SVRAT/SVMAX/SVMIN}$

$\text{$ putsys(6,2) $ reset to normal output unit 6}$

$\text{ENDIF $ BAD}$

$\text{PUTSYS(1,209) $ RESET SYSTEM CELL 209 TO DEFAULT}$

$\text{RETURN $}$

$\text{END $ CHKAJJT}$
Listing 2. - Sample Input Data File for 15° Swept Wing Constant Chord -Subsonic Model

nastran mesh
assign userfile='15d-045-24b.f25' unit=25 form=formatted delete new
ID MSC, chk-ajjt $ EDB - 27 Oct 1999
$$$$$$$$ FIFTEEN SWEEP $$

MODEL DESCRIPTION $ MODEL A OF NACA RM L55E11 $
15 DEGREE SWEEP WING $ QUAD4 MODEL $
SOLUTION KE FLUTTER ANALYSIS METHOD$ USING DOUBLET LATTICE METHOD$ AERODYNAMICS $
RUN PRODUCES XY PLOTS OF THE V-G FLUTTER DATA$ AND STRUCTURE PLOTS $
$$$$$$$$
TIME 10 $ diag 8,56
SOL 145 $ FLUTTER ANALYSIS include 'ajjt-chka.v705'
CEND
TITLE = 15-DEG SWEEP WING (DLM AERODYNAMICS) 4 CHORDWISE BOXES $ chk-ajjt
SUBT = MACH 0.45 QUAD4 Plate model
LABEL = KE METHOD FLUTTER SOLUTION
ECHO = SORT
SPC = 1 $ WING ROOT FIXED
METHOD = 1 $ LANCZOS
Cmethod = 20 $ HESS
FMETHOD = 30 $ KE-FLUTTER METHOD
SET 1 = 1 THRU 124 $ PHYSICAL GRIDS
DISP(PLOT) = 1 $
OUTPUT(PLOT)
CSCALE 2.0
PLOTTER NASTRAN
SET 1 = AERO1,QUAD4
SET 2 = QUAD4
VIEW 34.,23.,0.
PTITLE = STRUCTURAL ELEMENTS
FIND SCALE, ORIGIN 1, SET 2
PTITLE = STRUCTURAL AND AERODYNAMIC ELEMENTS
FIND SET 1
PLOT MODAL 0 ORIGIN 1, SET 1 SYMBOL 6 VECTOR R
VIEW 0.,90.,0.
FIND SCAL E, ORIGIN 1,SET 1
CONTOUR ZDISP
PLOT MODAL 0 CONTOUR OUTLINE ORIGIN 1, SET 2
OUTPUT (XYOUT)
CSCALE 2.0
PLOTTER NASTRAN
CURVELINESYMBOL = -6
YTTITLE = DAMPING G
YBTITLE = FREQUENCY F Hz
XTITLE = VELOCITY V (in/sec)
XTGRID LINES = YES
XBGGRID LINES = YES
YGRID LINES = YES
YBGGRID LINES = YES
UPPER TICS = -1
TRIGHT TICS = -1
BRIGHT TICS = -1
XYPLOT VG / 1(G,F) 2(G,F) 3(G,F) 4(G,F) 5(G,F) 6(G,F)
BEGIN BULK

$param,chkajjt,yes
$param,ceigajjt,yes
$param,post,0

**$ param,chkajj,t,yes**

**$ param,ceigajj,t,yes**

**$ param,post,0**

**$ eigc 20 hess max 24**

***

**$ EGRID 11 -1.0353 0. 0.**

**$ EGRID 12 .44517 5.5251 0.**

**$ EGRID 13 2.5157 5.5251 0.**

**$ EGRID 14 1.0353 0. 0.**

**$ GRIDG 1 -11 -12 -13 +GG1**

**$ LIST 5 .25 .5 .5 .5 .25**

**$ GRIDU 1 1 THRU 78**

***

**$ EGRID 11 -1.0353 0. 0.**

**$ EGRID 12 .44517 5.5251 0.**

**$ EGRID 13 2.5157 5.5251 0.**

**$ EGRID 14 1.0353 0. 0.**

**$ GRIDG 1 -11 -12 -13 +GG1**

**$ LIST 5 .25 .5 .5 .5 .25**

**$ GRIDU 1 1 THRU 78**

***

**$ CORD2R 1 .0 .0 .0 .0 .0 1. +C1**

**$ comment out the next 3 lines for free model**

**$ SPC1 1 4 14 53**

**$ SPC1 1 12356 14 27 40 53**

**$ SPC1 1 16 1 THRU 78**

**$ SPC1 1 6 1 THRU 13**

**$ SPC1 1 6 15 THRU 26**

**$ SPC1 1 6 41 THRU 52**

**$ SPC1 1 6 54 THRU 78**

**$ uncomment the next 6 lines for free model**

**$ SPC1 1 126 99**

**$ suport 99 345**

**$ GRID 99**

**$ RBE2 99 99 123456 14 27 40 53**

**$ CONM2 90 99 1.4 +1.**

**$ 1.4 1.4 1.4**

***

**$ CGEN QUAD4 1 1 1 1 12 +LE**

**$ +LE .000 .000 .312 .312**

**$ CGEN QUAD4 1 1 1 13 48**

**$ CGEN QUAD4 1 1 1 49 60 +TE**

**$ +TE .312 .312 .000 .000**

**$ PSHELL 1 1 .041 1 1**

***

**$ MAT1 1 10.4+6 3.9+6 2.61-4 ALUMINUM**

**$ PARAM COUPMASS1**

***

**$ AERO ACSID VELOCITY REFC RHOREF SYMXZ SYMXY**

**$ AERO 0 2.0706 1.1092-7 1**

**$ CAERO1 EID PID CP NSPAN NCHORD LSPAN LCHORD IGID +CONT**

**$ CAERO1 101 1 1 6 4 +CA101**

**$ +CA101 -1.0 -.26795 0. 2.0706 -1. 5.45205 0.0 2.0706**

**$ SCAERO1 M1 M2 M3 M4 M5 M6 M7 M8 +CONT**

**$ SCAERO1 1 .45 .45 .45 .45 .45 .45 .45**

**$ MKAERO1 .45 .45 .45 .45 .45 .45 .45 .45**

**$ +MK .001 .025 .05 .075 0.1 .125 0.15 0.175**

**$ MKAERO1 .45 .45 .45 .45 .45 .45 .45 .45**

**$ +MKA 0.2 0.25 0.3 0.35 0.4 0.45 0.5 0.55**

**$ MKAERO1 .45 .45 .45 .45 .45 .45 .45 .45**

**$ +MKA1 0.6 0.65 0.7 0.75 0.8 0.85 0.9 0.95**

**$ MKAERO1 .45 .45 .45 .45 .45 .45 .45 .45**

**$ +MKA2 1.0 1.1 1.2 1.3 1.4 1.5 1.6 1.7**

**$ MKAERO1 .45 .45 .45 .45 .45 .45 .45 .45**
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Toulouse, France

+MKA3  1.8  1.9  2.0  2.1  2.2  2.3  2.4  2.5
MKAERO1 .45  +MKA4
+MKA4  2.6  2.7  2.8  2.9  3.0  3.1  3.2  3.3
MKAERO1 .45  +MKB
+MKB  3.4  3.5  3.6  3.7  3.8  3.9  4.0  4.1
MKAERO1 .45  +MKBB
+MKBB  4.2  4.3  4.4  4.5
MKAERO1 .45  +MKB1
+MKB1  4.6  4.7  4.8  4.9  5.0  5.1  5.2  5.3
MKAERO1 .45  +MKB2
+MKB2  5.4  5.5  5.6  5.7  5.8  5.9  6.0  6.5
MKAERO1 .45  +MKB
+MKB  7.0  7.5  8.0  8.5  9.0  10. 12. 14.
$PAERO1 PID B1 B2 B3 B4 B5 B6
$SET1 SID G1 G2 G3 G4 G5 G6 --ETC.--+CONT
SET1  100  1  5  9  13  27  31  35 +S1
+S1  39  66  70  74  78
$SET1  100  1  THRU  26  27  THRU  78
$SET1  100  1  thru  13  14  thru  26  27
+S1  $  28  thru  39  40  thru  78
$param opkgkg 0
$param opgeom 0
$SPLINE1 EID CAERO BOX1 BOX2 SETG DZ
SPLINE1 100 101 101 124 100 .0
$***            ***$
$*** 15 DEG SWEPT WING EIGENVALUE AND FLUTTER CONTROL DATA    ***$
$***            ***$
$param OPPHIPA 1
ASET1  3  1  THRU  13
ASET1  3  15  THRU  26
ASET1  3  28  THRU  39
ASET1  3  41  THRU  52
ASET1  3  54  THRU  65
ASET1  3  66  THRU  78
eigr1  1  6
EIGR  10  MGIV  6  +ER
+ER  MAX
$FLFACT SID F1 F2 F3 F4 F5 F6 F7 +CONT
FLFACT  1  1.0
FLFACT  2  .45
$FLFACT  3  .2  .16667 .15315 .14286 .12500 .11111 .10000 KFREQ
$FLUTTER SID METHOD DENS MACH RFREQ IMETH NVALUE EPS
FLUTTER  30  KE  1  2  3  L  6
PARAM LMODES 6
ENDDATA

Figure A-1. - Subsonic $\text{Re}(Q_{hi})$ Matrix Convergence with Chordwise Aerodynamic elements as a function of Reduced Frequency.

Figure A-2. - Subsonic $\text{Im}(Q_{hi})$ Matrix Convergence with Chordwise Aerodynamic elements as a function of Reduced Frequency.
Figure A-3. - Supersonic Re(Qhh) Matrix Convergence with Chordwise Aerodynamic elements as a function of Reduced Frequency.